Complete & Orthogonal Replication of Hyperdimensional Memory via Elementary Cellular Automata

Nathan McDonald & Richard Davis

Air Force Research Lab/ Information Directorate, Rome, NY, USA; nathan.mcdonald.5@us.af.mil, PA: 88ABW-2019-1445

***Abstract*: Hyper-dimensional computing (HDC)/ Vector Symbolic Architectures (VSA) [1] implements associative learning using very large binary vectors. This approach has been used to model the learning and transfer learning of a foraging honeybee [2]. In real-world systems, we may want to copy learned associations onto multiple agents, e.g. swarm systems; however, simply copying the memory vectors across multiple agents makes all agents vulnerable to the same attack by a malicious entity. Therefore the challenge is to replicate the parent agent's item memory and compositional memory such that all learned associations are preserved yet the clone's memory vectors are maximally uncorrelated with the parent's memory vectors. This work evaluated all 256 elementary cellular automata (ECA) rules for this task and identified 8 rules that satisfied these replication requirements. To the best of the authors' knowledge, this is the first report of complete and orthogonal replication of HDC memory using ECA.**

Keywords: hyperdimensional computing, vector symbolic architectures, elementary cellular automata

BACKGROUND

In this work, HDC vectors are represented as vectors of '0's and '1's of length d = 1e4. The similarity between any two such vectors is measured by the Hamming distance (HD), the fraction of non-identical bits, e.g. for two random HDC vectors HD~0.5. Creating associative memories among HDC vectors typically involves three operations: binding, bundling, and cyclic shifting [1]. Bundling is a majority bit operation on a collection of vectors, denoted as [**a**+**b**+**c**]. Binding is analogous to assigning or reading a variable value and is performed as bitwise XOR, denoted as ⊗. Cyclic shift of vector **a** by j elements is denoted as Sh(**a**,j). In the canonical example, querying the compositional (associative) memory with "What is the dollar of Mexico?" returns a noisy version (0<HD<<0.5) of the encoded "peso" HDC vector [1].

ECA are cell-based binary state machines which follow a homogeneous rule for state transitions based on strictly local interactions [4]. Despite their simplicity, chaos and Turing complete behavior are demonstrated amongst these rules [4].

In [3], ECA were proposed to replicate HDC vectors such that the resultant clones were orthogonal, that is maximally uncorrelated (HD~0.5), to the parent vectors while still preserving (HD~0) their encoded associations through binding, bundling, and cyclic shift. An ECA rule is viable when for each parent HDC vector, clones (denoted *) resultant from the ith iteration of ECA rule R satisfy the target HDs (Tables 1 and 2) [2]. In [2], only ECA rule 90 was studied, and it only met Table 1 criteria.

METHOD

This work explored the viability of all 256 ECA rules to replicate HDC vectors, producing orthogonal clones while preserving the associations of all 3 HDC encoding operations. Random HDC vectors **a**, **b**, and **c** of length d = 1e4 were generated and operated upon according to Tables 1 and 2. The resultant Hamming distances were then calculated. Each ECA rule was tested for i = 1-50 iterations to study its cloning ability over time.

Table 1. HDC Binding & Shift Operations and Target HD

x	y	HD(x,y)
a	**a***	0.5
a⊗b	(**a⊗b**)*	0.5
Sh(**a**,10)	Sh(**a**,10)*	0.5
(**a⊗b**)*	**a*⊗b***	0
Sh(**a**,10)	Sh(**a***,10)	0

Table 2. HDC Bundling & Shift Operation and Target HD

x	y	HD(x,y)
[**a** + **b** + **c**]*	[**a*** + **b*** + **c***]	0
[**a⊗b** + **b** + **c**]*	[**a*⊗b*** + **b*** + **c***]	0
[**a** + **b** + Sh(**c**,10)]*	[**a*** + **b*** + Sh(**c***,10)]	0
[**b** + [**a** + **b** + **c**] + **c**]*	[**b*** + [**a*** + **b*** + **c***] + **c***]	0
[**a⊗b** + **b** + **c**]*	[**a*⊗b*** + **b*** + **c***]	0

RESULTS

8 ECA rules, viz. 15, 85, 154, 166, 170, 180, 210, and 240, achieved all target HDs. While rules 170 and 240 satisfied conditions with every iteration (Fig. 1a-b), the remaining six rules were viable only every 8th iteration (Fig. 1c-d) as indicated by the periodic striping. Each of these 8 rules when expressed as a Boolean cube are related to one another, either of the Literal or MUX-XOR logic family, accounting for their utility in this task [5].

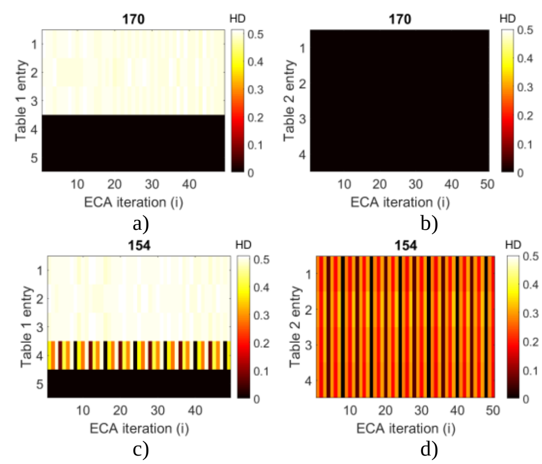

Fig. 1 HD between parent HDC vectors and clones via ECA rule a-b) 170 and c-d) 154 according to Tables 1 and 2 criteria.

All 256 ECA rules were then tested on a foraging bee associative learning task (see Supplementary Materials), and only the 8 aforementioned rules demonstrated complete replication (SM: Fig 1a,b red circles) and orthogonality (not shown) of the learned associations.

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