Evaluating One-Dimensional Continuous & Discrete Heterotic Systems

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In order to characterise computational systems, for example different proposed substrates for Reservoir Computing (RC), measures such as memory capacity, kernel rank, and generalisation capacity have been created**.** The broad applicability of the reservoir computing paradigm to many different dynamical systems has recently lead to the development of a framework for the Characterisation of Reservoir Computers (ChaRC), for comparing between substrates using these measures [Dale et al 2018, 2019]. ChaRC exploits several computational measures to assess the ability of a substrate to realise different reservoirs, and therefore its capacity as a generic RC substrate. However it is not yet clear how ChaRC may be applied to discrete state systems such as Cellular Automata or Random Boolean Networks.

To investigate discrete state space reservoirs, we take as examples two analogous systems: Threshold Coupled Maps (TCMs) [Sinha & Ditto 1998] (coupled logistic maps) and one-dimensional cellular automata (CA) [Wolfram 1983]. These two systems are comparable in their 1D topologies, and their discrete time behaviour. However, TCMs have continuous-valued state variables, whereas CAs have discrete valued variables.

TCMs and their ease of simulation provide a benchmark for many measures of spatio-temporal chaos such as Lyapunov spectra [Isola et al 1990], Kolmogorov-Sinai entropy density [Batista & Viana 2002], and Mutual Information and correlation in space-time [Schrieber 1990]. These are potential behaviour measures for use by ChaRC. Given TCMs similarities to some aspects of CAs, we determine how to transfer ChaRC-related measures to the discrete domain.

We also use these systems to look for insights into characterising *heterotic* computing systems [Kendon et al 2015]. We compare *quasi-uniform cellular automata* [Gundersen 2017] in which different regions of cells contain different rules, and TCMs in which each map possesses a different parameter value. By investigating the analogy between these two systems we determine a method of applying ChaRC to heterotic continuous and discrete state systems.

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